MATH COMPETENCY PRACTICE TEST ANSWERS EXPLAINED

Perform the indicated operation:

1. $\frac{7}{10} \cdot \frac{15}{2} \cdot \frac{3}{14}$

To simplify, you can cancel, where possible, and then multiply.

$\frac{\cancel{7}^{1}}{\cancel{10}_{2}} \cdot \frac{\cancel{15}^{3}}{2} \cdot \frac{3}{\cancel{14}_{2}} = \frac{9}{8}$

Or multiply, without cancelling, and then reduce the fraction.

$\frac{7}{10} \cdot \frac{15}{2} \cdot \frac{3}{14} = \overset{(5)}{\frac{315}{280}} = \overset{(7)}{\frac{63}{56}} = \frac{9}{8}$

2. $\frac{7}{15} + \frac{1}{18}$

In order to add or subtract fractions, there must be a common denominator. Find the lowest number that both 15 and 18 will divide into. That number is 90. 15 divides into 90, 6 times and 6 times 7 is 42. The first fraction becomes $\frac{42}{90}$

Second fraction: 18 divides into 90 = 5 times and 5 times 1 is 5. The second fraction becomes $\frac{5}{90}$

Now the denominators are the same and we can add. 42 plus 5 is 47. The answer is $\frac{47}{90}$

3. $1\frac{1}{4} \div 2\frac{2}{3}$

First, turn each mixed number into a fraction. 4 times 1 is 4 plus 1 is 5. The first mixed number becomes $\frac{5}{4}$

Second mixed number: 3 times 2 is 6 plus 2 is 8. The second mixed number is $\frac{8}{3}$

Now the problem is: $\frac{5}{4} \div \frac{8}{3}$

When dividing fractions, flip the second fraction and then multiply: $\frac{5}{4} \cdot \frac{3}{8} = \frac{15}{32}$

4. $10\frac{1}{4} - 5\frac{3}{5}$

The problem asks you to subtract fractions, so you must find a common denominator, BUT FIRST turn each mixed number into a fraction.

4 times 10 is 40 plus 1 = $\frac{41}{4}$

5 times 5 is 25 plus 3 = $\frac{28}{5}$

Now, what is the smallest number that both 5 and 4 will divide into? 20, of course.

4 divided into 20 is 5 times 41 = $\frac{205}{20}$

5 divided into 20 is 4 times 28 = $\frac{112}{20}$

$\frac{205}{20} - \frac{112}{20} = \frac{93}{20}$

To complete the problem, divide 20 into 93 and the answer is $4\frac{13}{20}$

5. 0.002 x 1.4

.002
x 1.4
008
002
.0028

(The zero before the decimal is irrelevant.)

What are the total number of points after the decimals in the problem? There must be that many points after the decimal in the answer.

6. Divide 0.00578 by 1.7

$$1.7\overline{).00578}\quad .0034$$

Calculate and simplify:

7. -3 - (-4) + 5

When subtracting signed numbers, change the sign of the number being subtracted and then follow the rules for addition of signed numbers.

-3 + 4 + 5

When adding signed numbers, if the signs are different, find the difference in the two numbers and use the sign of the biggest number. If the signs are the same, add-up the numbers and give your answer the sign.

-3 and +4 is +1 and +1 and +5 = 6 .

8. (-2)(-3) - (5)(-2)

First do the multiplication. When multiplying or dividing, if the signs are the same, the answer is positive. If the signs are different, the answer is negative.
After multiplying, the problem is 6 - (-10). Change the sign of the number being subtracted and then follow the rules for addition.

6 + 10 = 16

9. $2\frac{1}{4} - 1\frac{1}{3} - 2\frac{1}{5}$

See instructions for #4. $\frac{9}{4} - \frac{4}{3} - \frac{11}{5}$

Lowest common denominator is 60. $\frac{135}{60} - \frac{80}{60} - \frac{132}{60} = -\frac{77}{60}$

10. $\frac{x^4 \cdot x^5}{x \cdot x^2}$

First do the multiplication. Remember, when multiplying, add the exponents.

$\frac{x^9}{x^3}$

Now divide. Remember, when dividing, subtract the exponents. Answer is x^6

Don't forget, the x in the problem was really x^1

11. 2x - (-x - y) - y

Remember the rule for subtracting signed numbers. When we change the signs of the numbers being subtracted, we get, 2x + x + y - y
The +y and -y make 0, so the answer is 3x.

12. Evaluate $b^2 - 4ac$
when $a = -3$
$b = -2$
$c = 3$

For this problem, simply plug in the numbers, and the problem becomes

$4 - 4(-3 \cdot 3)$
$4 - 4(-9)$
$4 - (-36)$
$4 + 36 = 40$

Solve:

13. $-2t + 1 = -5$
The goal of this problem is to disover what t equals. I need to get all variables on one side of the equation and all numbers on the other side of the equation. I know that I can do anything to one side of the equation as long as I do EXACTLY the same thing to the other side of the equation. The first thing I want to do is get that +1 out of the left side of the equation. If I subtract 1 from that 1, I get 0 and thus remove it from the left side of the equation. But wait . . . I must do the same thing to the other side of the equation. -5 and -1 are -6. Now the problem reads, $-2t = -6$
The next step is to remove the -2. Since it is a multiplier, I can do the opposite (divide) and remove it. It is important to remember that I am dividing by -2 not just 2. So, if I divide -2t by -2, I am left with just t. Now I divide the other side of the equation by -2 and my answer is $t = 3$

14. $22 = \frac{3x}{4} - 2$

Remember, I want to get all the variables on one side of the equation and all the numbers on the other side. To remove the -2 from the right side of the equation, I add +2 and also add +2 to the left side of the equation.
Now the problem reads $22 + 2 = \frac{3x}{4}$ So $24 = \frac{3x}{4}$

To get rid of the 4 divisor, I do the opposite of division and multiply both sides of the equation by 4.
Now the problem reads
$96 = 3x$
To get rid of the 3 multiplier, I do the opposite and divide both sides by 3.
The answer is $32 = x$

15. $4(d - 1) = 3(d + 1)$

First, do the multiplication, and the problem reads $4d - 4 = 3d + 3$
Next, get rid of the -4 on the left side by adding it to both sides.
Now, the problem reads $4d = 3d + 7$
Finally, move the 3d from the right side of the equation by subtracting it from both sides.
We are left with $d = 7$

16. $(4x + 2)(5x + 7)$

Use FOIL to multiply and the answer is $20x^2 + 38x + 14$

17. A 200-foot piece of rope is to be cut into four pieces such that the first two pieces are the same length, the third piece is twice the first piece, and the fourth piece is three times the third piece. How long is each piece of rope?

Let x = 1st piece
Let x = 2nd piece (The first and second piece are the same length.)
Let $2x$ = 3rd piece (It's twice as long as the first piece.)
Let $3(2x)$ = 4th piece (It's three times the length of the third piece.)

Now we have an equation. All of these pieces added together equal 200 feet, SO . . . .

$$\begin{aligned} x + x + 2x + 3(2x) &= 200 \\ 10x &= 200 \\ x &= 20 \end{aligned}$$

So, the first two pieces are 20 feet each, the third piece is 2 times 20 or 40 feet, and the fourth piece is 3 times 40 or 120 feet.

18. Joe and Dave had a roofing business. Joe, as owner of the materials, received 3 dollars for every 2 dollars Dave received. On a job that paid \$750, what amount did each receive?

Let x = \$1
Joe gets \$3 or $3x$ and Dave gets \$2 or $2x$. Together they get \$750, SO . . .

$$\begin{aligned} 3x + 2x &= \$750 \\ 5x &= \$750 \\ x &= \$150 \end{aligned}$$

Joe gets 3 times \$150 or \$450 and Dave gets 2 times \$150 or \$300.

19. Multiply $x^2 - 2x - 3$ by $x + 2$
Pay close attention to the signs of the numbers as you multiply.

$$\begin{array}{r} x^2 - 2x - 3 \\ \underline{\quad x + 2} \\ 2x^2 - 4x - 6 \\ \underline{x^3 - 2x^2 - 3x\quad} \\ x^3 \quad - 7x - 6 \end{array}$$

The $+2x^2$ and $-2x^2$ cancel each other out, so the answer is $x^3 - 7x - 6$

Proportions:

20. $\frac{8}{3} = \frac{24}{x}$

Cross multiply. 8 times x is 8x and 3 times 24 is 72. SO . . . $8x = 72$ . . . $x = 9$

21. On a map of the United States, 2 inches represent 650 miles. How many inches of map distance represent 1,625 miles?

This problem is solved using proportion. 2 is to 650 as x is to 1625.

$\frac{2}{650} = \frac{x}{1625}$

Cross multiply. 650 times x is 650x and 2 times 1,625 is 3,250. $650x = 3250$

$x = 5$

The answer is 5 inches.

Solve these Inequalities:

We solve inequalities in exactly the same way we solve equations. There is just one extra thing to remember. We can multiply or divide an equality by a negative number, but, when we do, we must change the direction of the inequality in the answer.

22. $-2a > 12$

$a < -6$

23. $s - 8 \leq 6$

$s \leq 14$

24. $9m + 7 > 43$

$9m > 36$

$m > 4$

Factor:

25. $2t^2 - 32$

Is there a number that will divide into each of these terms? Yes, 2 will divide into each, so 2 is one factor. After dividing by 2, we are left with $t^2 - 16$. $t^2 - 16$ falls under the "difference of two squares" rule. Both t^2 and 16 are the result of numbers multiplied times themselves - they have square roots.
$t^2 - 16 = (t - 4)(t + 4)$
Therefore, the factors of $2t^2 - 32$ are $2, (t - 4), (t + 4)$

26. $6x^2 - 7x - 3$

For this problem we are factoring a trinomial. We start by assuming that the trinimial is the product of two binomials $(? + ?)(? - ?)$ (The signs must be different in order for the last term to be a negative 3.)
Think back to the FOIL method of horizontal multiplication.
F stands for first, so look at the first term, $6x^2$. The first two terms in the binomial must be EITHER 3x and 2x or 6x and x. Let's try 3x and 2x.
$(3x + ?)(2x - ?)$

Next we skip to **L**, the last term of the trinomial. It is -3. What two numbers will multiply to make 3 and what will be their signs? Well, since the middle term of the trinomial is negative, there must also be a negative in one of the factors. We know there isn't a negative in both the factors because the **L**ast term in the trinomial (-3) is a negative.
Our possibilities are:
-1 and +3
+1 and -3.
Now it is a matter of trial and error to determine which pair to use and where.
First we will try $(3x - 3)(2x + 1)$

Use the FOIL method of multiplication to check this answer.
NOPE, that is wrong because, when multiplied, we get $6x^2 - 3x - 3$.

Let's try again. This time $(3x + 1)(2x - 3)$
YES!! That's the right combination!
Sure enough, the answer is $6x^2 - 7x - 3$

SO . . . . the factors of $6x^2 - 7x - 3$ are $(3x + 1)(2x - 3)$

27. $5x^3 - 125x$

Is there a number that will divide into both $5x^2$ and $125x$? Yes, there is. $5x$ will divide into both, so $5x$ becomes one factor. After dividing by $5x$, we are left with $x^2 - 25$. You know what?! We have run into the "difference of two squares rule" again. Both x^2 and 25 have square roots. They are the result of x squared and 5 squared. SO . . . the factors of $5x^3 - 125x$ are $5x, (x + 5), (x - 5)$

28. Solve the equation: $x^2 - 9 = 0$
First we add 9 to the left side, which gets rid of the 9 there and then add 9 to the right side, and our equation now reads $x^2 = 9$
Next we use factoring to solve this equation:
What numbers, multiplied times themselves give us +9. Well, 3, of course, but also -3.

So $x = 3, -3$

Graph:

29. $y = -2x + 4$

X	Y
+1	
+3	
+4	

30. $2y = 4x - 6$

X	Y
+1	
+2	
+3	

These two problems are really just the process of solving for two unknowns when the values are stated for one of the unknowns.

First create a values chart (fill in those spaces under the y). To do that for problem #29, we need to find out the value for y when the value of x is +1, when the value of x is +3, and when the value of x is +4.

If the value of x is +1 and we plug that value into the equation, then: $y = -2(+1) + 4$
$y = -2 + 4$
$y = +2$

So, when x has a value of +1, y has a value of 2.

Next we solve the equation when the value of x is +3: $y = -2(+3) + 4$
$y = -6 + 4$
$y = -2$

So, when x has a value of +3, y has a value of –2.

Next we solve the equation when the value of x is +4. $y = -2(+4) + 4$
$y = -8 + 4$
$y = -4$

So, when x has a value of +4, y has a value of –4.

The next step is to write each of the of the x and y coordinates (values) as an order pair – the x coordinate always goes first, so the coordinates are: (+1, +2) (+3, -2) (+4, -4)

Now plot these points on the graph. The first point is where the +1 on the x line meets the +2 on the y line, and so on. Then draw a line through the points. Your graph will look like this:

29.